Joint modelling of multiple treatment variables for a single outcome: A Bayesian approach

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MOTIVATION

OBJECTIVES

1. The current frameworks for causal inference in observational studies **do not readily allow for multiple mixed type treatment variables**, such as continuous, discrete, ordinal, and ordered categorical variables
2. We propose the extended rank likelihood (ERL) method[1] to **estimate propensity scores on the latent level**[2] — to allow the simultaneous inclusion of diverse types of treatment variables

APPLICATION

- Multiple treatments are useful both in medical studies and policy-making for assessing how two treatments jointly affect the outcome
- **Multiple treatments** here is not to be confused with *multi-valued treatment* (modelled under a multinomial distribution) or *multiple versions of treatment* (same variable type but different versions)
- Case studies with our:
  - **Latent propensity function (LPF)**, applied to the 1987 National Medical Expenditure Survey (NMES) data[3]
  - **Latent generalised propensity score (LGPS)**, applied to the LAtent Causal Socioeconomic Health (LACSH) data

EXTENDED RANK LIKELIHOOD[1]

WHAT IS IT?

- An approach to copula estimation in which the marginal distributions are arbitrary and of unspecified types, thus **accommodating both discrete (ordinal) and continuous data**
- Built on **Sklars theorem**: Multivariate distribution = univariate marginal distribution + copula
- We **only rely on the ranks of the data,** suitable for variables (e.g. maternity leave days in the case study) that are not truly continuous

APPLICATION OF ERL

MULTIPLE TREATMENTS

Suppose we have K treatment variables of different structures, e.g. continuous and ordinal and P observed covariates.

Let $T_1, \dots, T_K$ be the treatments on the observable scale; $U_{T_1}, \dots, U_{T_K}$ be the *latent estimates*; $X_1, \dots, X_P$ be the covariates on the observable scale; $U_{X_1}, \dots, U_{X_P}$ be the *latent estimates*;

$$
\begin{aligned}
T_{i,k} &= F_k^{-1}[\Phi(u_{T_{i,k}})]; \quad k = 1, \dots, K \\
X_{i,p} &= F_p^{-1}[\Phi(u_{X_{i,p}})]; \quad p = 1, \dots, P \\
u_i &= \left(u_{T_{i,1}}, \dots, u_{T_{i,K}}, u_{X_{i,p}}, \dots, u_{X_{i,P}}\right)' \\
\boldsymbol{u}_1, \boldsymbol{u}_2, \quad \dots, \quad \boldsymbol{u}_N | \boldsymbol{C} &\overset{i.i.d.}{\sim} MVN(0, \boldsymbol{C})
\end{aligned}
$$

where F is estimated by the empirical CDF; conditioned on the F's, we estimate the u's and C in a Bayesian framework.

LATENT PROPENSITY SCORES FOR MULTIPLE TREATMENTS

On the latent level, one can consider various versions of propensity scores. We consider two versions:

1. Latent Generalised Propensity Scores

$$
\begin{aligned}
U_R &= r(u_{T_{i,1}}, \dots, u_{T_{i,K}}, u_{X_{i,p}}, \dots, u_{X_{i,P}}, C) \\
&= \Phi_K(U_T;\ C_{TX} C_{XX}^{-1} U_X, C_{TT} - C_{TX} C_{XX}^{-1} C_{XT})
\end{aligned}
$$

2. Latent Propensity Function

$$
\begin{aligned}
e\{\cdot | U_\theta\} &= E[U_T | U_X] \\
&= C_{TX} C_{XX}^{-1} U_X
\end{aligned}
$$

APPLICATION OF LPF TO THE NMES DATA

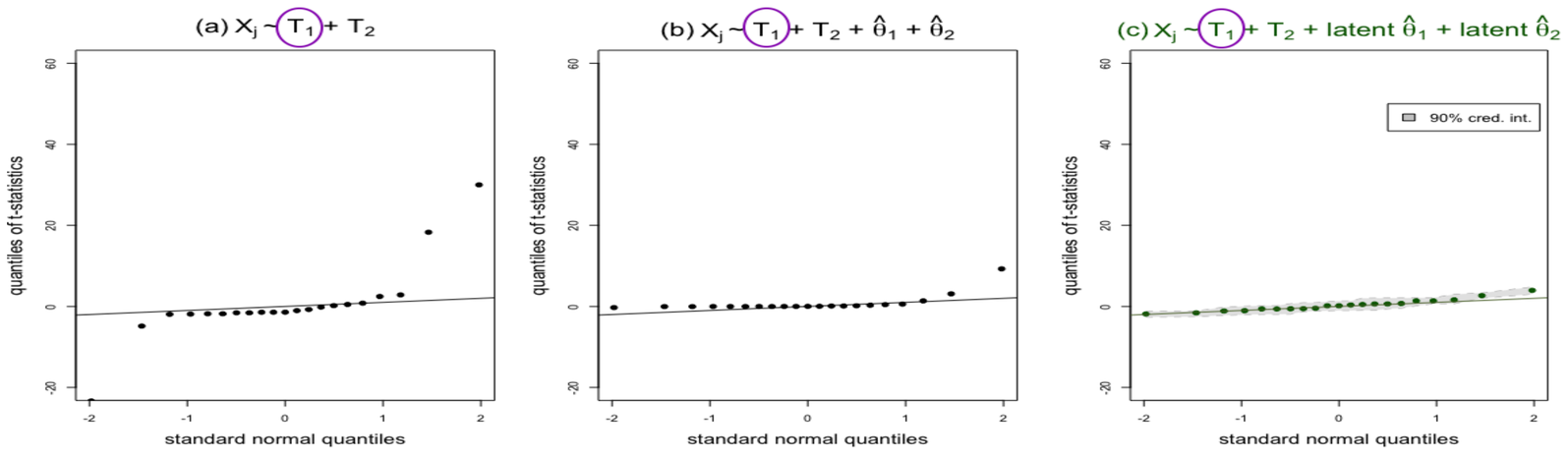

FIG. 1: CHECKING ASSUMPTIONS: Standard normal quantile plots of t-statistics for T_1 = *duration of smoking* against each covariate, (a) without controlling for propensity scores, (b) controlling for the traditional PF, (c) controlling for our LPF estimated from the ERL

APPLICATION OF LGPS TO THE LACSH DATA

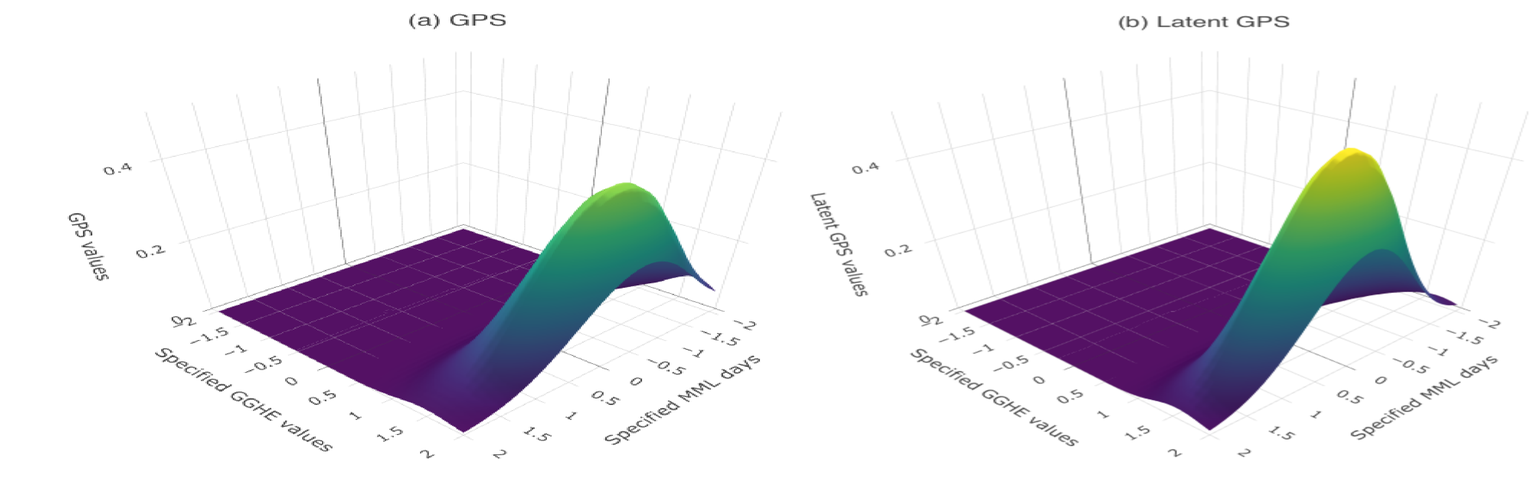

FIG. 2: Posterior median function for (a) GPS assuming bivariate normal treatments and (b) Latent GPS using ERL computed based on Step 4 under DOSE-RESPONSE SURFACE. The figures are for a selected country (Australia) from that LACSH data given treatment (*MML* and *GGHE*) counterfactual values

DOSE-RESPONSE SURFACE

FOR DUAL TREATMENTS ON LACSH DATA

Using the LACSH data (latent health of countries), we obtain a dose-response surface (expected health of country) with two specified treatment variables:

- Mandatory Maternity Leave (MML) days *(numerical but in fact more ordinal)* and
- General Government Health Expenditure (GGHE) *(continuous)*

PROCEDURE:

1. Use treatment and covariate data to estimate $U_{T_1}, U_{T_2}, U_{X_1}, U_{X_2}, \dots, U_{X_p}$ through ERL
2. Compute generalised propensity scores U_R on the latent level
3. Fit regression to obtain $\hat{\beta}$'s: $\mu_H(T_1, T_2, U_R) = \beta_0 + \beta_1 T_1 + \beta_2 T_2 + \beta_3 U_R + \beta_4 T_1 T_2 U_R$
4. Specify counterfactual values T_1^* and T_2^* to compute $U_{T_1^*}$, $U_{T_2^*}$, and U_R^*
5. Compute $\mu_H^* = \mu_H(T_1^*, T_2^*, U_R^*)$ using the $\hat{\beta}$'s from (3)
6. Plot μ_H^* against T_1^* and T_2^*

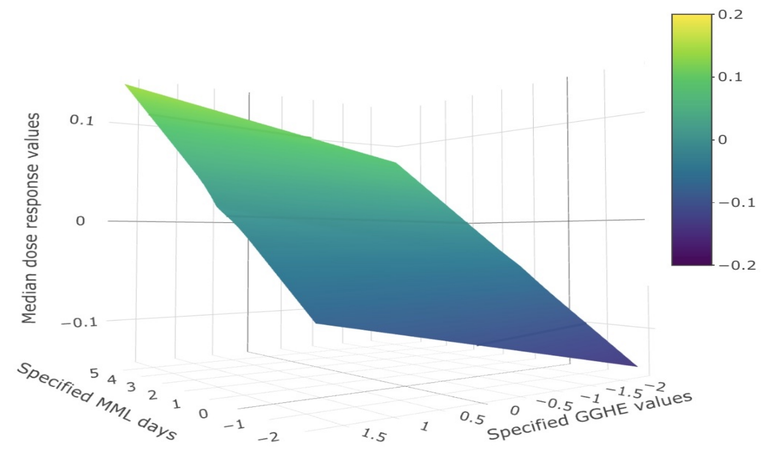

FIG. 3: Posterior median dose-response surface for dual treatment

We are able to produce the posterior uncertainty of the dose-response surface (not visualised here).

REFERENCES

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[3] K Imai and D. A. Van Dyk. Causal inference with general treatment regimes: Generalizing the propensity score. *JASA*, 2004.